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Next-to-Leading order approximation of polarized valon and parton distributions

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Abstract: Polarized parton distributions and structure functions of the nucleon are analyzed in the improved valon model. The valon representation provides a model to represent hadrons in terms of quarks, providing a unified description of bound state and scattering properties of hadrons. Polarized valon distributions are seen to play an important role in describing the spin dependence of parton distributions in the leading order (LO) and next-to-leading order (NLO) approximations. In the polarized case, a convolution integral is derived in the framework of the valon model. The Polarized valon distribution in a proton and the polarized parton distributions inside the valon are necessary to obtain the polarized parton distributions in a proton. Bernstein polynomial averages are used to extract the unknown parameters of the polarized valon distributions by fitting to the available experimental data. The predictions for the NLO calculations of the polarized parton distributions and proton structure functions are compared with the LO approximation. It is shown that the results of the calculations for the proton structure function, xg_1^p, and its first moment, Γ_1^p, are in good agreement with the experimental data for a range of values of Q^2. Finally the spin contribution of the valons to the proton is calculated.

Keywords: NLO Computations, Parton Model, Phenomenological Models.

Contents

1. Introduction

Determination of parton distributions in a nucleon in the framework of quantum chromodynamics (QCD) always involves some model-dependent procedure. Instead of relying on mathematical simplicity as a guide, we take a viewpoint in which the physical picture of the nucleon structure is emphasized. That is, we consider the model for the nucleon which is compatible with the description of the bound state problem in terms of three constituent quarks. We adopt the view that these constituent quarks in the scattering problems should be regarded as the valence quark clusters rather than point-like objects. They have been referred to as *valons*. In the valon model, the proton consists of two "up" and one "down" valons. These valons thus carry the quantum numbers of the respective valence quarks. Hwa [1] found evidence for the valons in the deep inelastic neutrino scattering data, suggested their existence and applied it to a variety of phenomena. Hwa [2] has also successfully formulated a treatment of the low-p_T reactions based on a structural analysis of the valons. Some papers can be found in which the valon model has been used to extract new information for parton distributions and hadron structure functions [3].

Hwa and Yang in two papers [4] refined the idea of the valon model and extracted new results for the valon distributions. In a recent paper, unpolarized parton distributions and hadronic structure functions in the NLO approximation were extracted [5]. The pion and meson structure functions in the valon model framework has also been analyzed [6, 7]. In this paper we extend the idea of the valon model to both polarized and unpolarized cases in the LO and NLO approximations.

The plan of the paper is as follows. In subsection 2.1, unpolarized valon distributions are explained. In subsection 2.2 we define a convolution integral, using a linear combination of spin-up and down quark distributions. In subsection 2.3 the relationship between unpolarized and polarized valon distributions in a proton is dealt with. The discussion of the polarized valons for the non-singlet and singlet cases is further refined in subsection 2.4. $\mathcal{W}$ functions which link the polarized valon distributions and the unpolarized ones are described in subsection 2.5. These contain some unknown parameters which can be obtained by fitting to the experimental data. In section 3 the moments of the polarized valon and polarized parton distributions are analyzed in the NLO approximation. Subsection 4.1 describes the method of ref. [8] in which one constructs averages of the measured polarized structure functions weighted by suitably chosen Bernstein polynomials. These experimental Bernstein averages are then fitted in subsection 4.2, using the CERN subroutine MINUIT, to the QCD predictions for the corresponding linear combinations of moments. In section 5 the inverse Mellin transformation and the convolution integral are used to calculate the polarized parton distributions in the x-space. In subsection 6.1 the polarized structure function, xg_1, for the proton is calculated in the LO and NLO approximations, and compared with the experimental data and some other theoretical models. Our prediction for the first moment of proton structure functions are presented in this subsection for some values of Q^2. The spin contribution of valons to the proton are discussed and calculated in subsection 6.2. We also give our conclusions in section 7.

2. Valon distributions in unpolarized and polarized cases

2.1 Valon distributions

The subject of nucleon structure has mainly been investigated from two opposite viewpoints. On the one hand, one studies the bound state in terms of three quarks, such as in a bag model, and obtains various static properties of the nucleon and its spectroscopic partners. On the other hand, one probes the nucleon with high energy leptons in the hope of learning something about its three constituent quarks but finds structure functions that can only be understood in terms of an infinite number of partons (i.e., quarks, antiquarks and gluons). The two views can be reconciled if we recognize that the constituents in the quark model for the static nucleon are not the same objects as the quarks in the parton model. Since the scales of spatial resolution are different for the two problems, we may regard the former as clusters of the latter without contradicting either view. The clusters have the quantum numbers of the valence quarks, and have been called valons, for short. More precisely, a valon can be defined as a valence quark and associated sea quarks and gluons which arise in the dressing processes of QCD. In a bound state problem these processes are virtual and a good approximation for the problem is to consider a valon as an integral unit whose internal structure cannot be resolved. In a scattering situation, on the other hand, the virtual partons inside a valon can be excited and be put on the mass shell. It is therefore more appropriate to think of a valon as a cluster of partons with some momentum distribution. The proton, for example, has three valons which interact with each other in a way that is characterized by the valon wave function, while they respond independently in an inclusive hard collision with a Q^2 dependence that can be calculated in QCD at high Q^2.

In the static problems there is a little difference between the usual constituent quarks and the valons, since the point-like nature of the constituent quarks is not a crucial aspect of the description, and has been assumed mainly for simplicity. But, in the scattering problems it is important to recognize that the valons, being clusters of partons, can not easily undergo scattering as a whole. The fact that the bound-state problem of the nucleon can be well described by three constituent quarks implies that the spatial extensions of the valons do not overlap appreciably. A physical picture of the nucleon in terms of three valons is then quite analogous to the usual picture of the deuteron in terms of two nucleons.

According to this description of the valon model, $F_2^p(x, Q^2)$ as a structure function (e.g. F_2 or xF_3) is related to a valon structure function $\mathcal{F}_2^v$ corresponding to a valon v, by smearing of the valon momentum in the nucleon. On the other hand, the structure function of a hadron is obtained by convolution of two distributions: the valon distributions in the proton and the structure function for each valon. In the unpolarized situation we may write:

$$F_2^p(x, Q^2) = \sum_v \int_x^1 dy G_{v/p}(y) \mathcal{F}_2^v(\frac{x}{y}, Q^2) \,, \tag{2.1}$$

where the summation is over the three valons. Here $G_{v/p}(y)$ indicates the probability for the v-valon to have momentum fraction y in the proton. We shall also assume that the three valons carry all the momentum of the proton. This assumption is reasonable provided that the exchange of very soft gluons is responsible for the binding. Eq. (2.1) involves also the assumption that in the deep inelastic scattering at high Q^2 the valons are independently probed, since the shortness of interaction time makes it reasonable to ignore the response of the spectator valons. Thus, through eq. (2.1) we have broken up the hadron structure problem into two parts. One part represented by $G_{v/p}(y)$, describes the wave functions of the proton in the valon representation. It contains all the hadronic complications due to the confinement. It is independent of Q^2 or the probe. The other part represented by $\mathcal{F}_2^v(\frac{x}{y}, Q^2)$, describes the virtual QCD processes of the gluon emissions and quark-pair creation. It refers to an individual valon independent of the other valons in the proton and consequently also independent of the confinement problem. It depends on Q^2 and the nature of the probe.

Hwa and Zahir [9] assumed a simple form for the exclusive valon distribution which facilitated the phenomenological analysis as follows

$$G_{UUD/p}(y_1, y_2, y_3) = g(y_1 y_2)^\alpha y_3^\beta \delta(y_1 + y_2 + y_3 - 1) \,, \tag{2.2}$$

where y_i is the momentum fraction of the i'th valon. The U and D type inclusive valon distributions can be obtained by double integration over the unspecified variables

$$\begin{aligned} G_{U/p}(y) &= \int dy_2 \int dy_3 G_{UUD/p}(y, y_2, y_3) \\ &= gB(\alpha+1, \beta+1) y^{\alpha}(1-y)^{\alpha+\beta+1}\,, \end{aligned} \tag{2.3}$$

$$\begin{aligned} G_{D/p}(y) &= \int dy_1 \int dy_2 G_{UUD/p}(y_1, y_2, y) \\ &= gB(\alpha+1, \alpha+1) y^{\beta}(1-y)^{2\alpha+1}\,. \end{aligned} \tag{2.4}$$

The normalization parameter g has been fixed by

$$\int_0^1 G_{U/p}(y)dy = \int_0^1 G_{D/p}(y)dy = 1\,, \tag{2.5}$$

and is equal to $g = [B(\alpha+1, \beta+1)B(\alpha+1, \alpha+\beta+2)]^{-1}$, where $B(m, n)$ is the Euler-Beta function.

R.C. Hwa and C.B. Yang [4] have recalculated the unpolarized valon distribution in the proton with a new set of parameters. The new values of α, β are found to be $\alpha = 1.76$ and $\beta = 1.05$.

2.2 Convolution integral

To describe the quark distribution $q(x)$ in the valon model, one can try to relate the polarized quark distribution functions $q^{\uparrow}$ or $q^{\downarrow}$ to the corresponding valon distributions $G^{\uparrow}$ and $G^{\downarrow}$. The polarized valon can still have the valence and sea quarks that are polarized in various directions, so long as the net polarization is that of the valon. When we have only one distribution $q(x, Q^2)$ to analyze, it is sensible to use the convolution in the valon model to describe the proton structure in terms of the valons. In the case that we have two quantities, unpolarized and polarized distributions, there is a choice of which linear combination exhibits more physical content. Therefore, in our calculations we assume a linear combination of $G^{\uparrow}$ and $G^{\downarrow}$ to determine respectively the unpolarized (G) and polarized (δG) valon distributions.

To relate $q^{\uparrow}$ and $q^{\downarrow}$ to both $G^{\uparrow}$ and $G^{\downarrow}$, we can consider linear combinations as follows

$$q^{\uparrow}_{i/p}(x, Q^2) = \sum_j \int_x^1 \left[\alpha\, G^{\uparrow}_{j/p}(y) q^{\uparrow\uparrow}_{i/j}\left(\frac{x}{y}, Q^2\right) + \beta\, G^{\downarrow}_{j/p}(y) q^{\downarrow\uparrow}_{i/j}\left(\frac{x}{y}, Q^2\right)\right] \frac{dy}{y}\,, \tag{2.6}$$

and

$$q^{\downarrow}_{i/p}(x, Q^2) = \sum_j \int_x^1 \left[\alpha'\, G^{\uparrow}_{j/p}(y) q^{\uparrow\downarrow}_{i/j}\left(\frac{x}{y}, Q^2\right) + \beta'\, G^{\downarrow}_{j/p}(y) q^{\downarrow\downarrow}_{i/j}\left(\frac{x}{y}, Q^2\right)\right] \frac{dy}{y}\,, \tag{2.7}$$

here $q^{\uparrow\uparrow}$ and $q^{\uparrow\downarrow}$ respectively denote the probability of finding q-up and q-down in a G-up valon and etc.

If we add and subtract the equations of (2.6), (2.7) we can determine the unpolarized and polarized quark distributions as follows

$$\begin{aligned}
q_{i/p}(x,Q^2) &\equiv q^{\uparrow}_{i/p}(x,Q^2) + q^{\downarrow}_{i/p}(x,Q^2) \\
&= \sum_j \int_x^1 \left(G^{\uparrow}_{j/p}\left[\alpha q^{\uparrow\uparrow}_{i/j} + \alpha' q^{\uparrow\downarrow}_{i/j}\right] + G^{\downarrow}_{j/p}\left[\beta q^{\downarrow\uparrow}_{i/j} + \beta' q^{\downarrow\downarrow}_{i/j}\right]\right) \frac{dy}{y}\,; \qquad (2.8) \\
\delta q_{i/p}(x,Q^2) &\equiv q^{\uparrow}_{i/p}(x,Q^2) - q^{\downarrow}_{i/p}(x,Q^2) \\
&= \sum_j \int_x^1 \left(G^{\uparrow}_{j/p}\left[\alpha q^{\uparrow\uparrow}_{i/j} - \alpha' q^{\uparrow\downarrow}_{i/j}\right] + G^{\downarrow}_{j/p}\left[\beta q^{\downarrow\uparrow}_{i/j} - \beta' q^{\downarrow\downarrow}_{i/j}\right]\right) \frac{dy}{y}\,. \qquad (2.9)
\end{aligned}$$

Since it is reasonable to assume that $q^{\uparrow\uparrow} = q^{\downarrow\downarrow}$ and $q^{\uparrow\downarrow} = q^{\downarrow\uparrow}$, to obtain the unpolarized and polarized quark distributions in the proton, we need to choose in eqs. (2.8), (2.9) $\alpha = \alpha' = \beta = \beta' = 1$.

Consequently we will have

$$q_{i/p}(x,Q^2) = \sum_j \int_x^1 \left(G^{\uparrow}_{j/p}[q_{i/j}] + G^{\downarrow}_{j/p}[q_{i/j}]\right) \frac{dy}{y}\,, \qquad (2.10)$$

and

$$\delta q_{i/p}(x,Q^2) = \sum_j \int_x^1 \left(G^{\uparrow}_{j/p}[\delta q_{i/j}] - G^{\downarrow}_{j/p}[\delta q_{i/j}]\right) \frac{dy}{y}\,, \qquad (2.11)$$

where $q_{i/j} \equiv q^{\uparrow\uparrow}_{i/j} + q^{\uparrow\downarrow}_{i/j} = q^{\downarrow\uparrow}_{i/j} + q^{\downarrow\downarrow}_{i/j}$ and $\delta q_{i/j} \equiv q^{\uparrow\uparrow}_{i/j} - q^{\uparrow\downarrow}_{i/j} = q^{\downarrow\downarrow}_{i/j} - q^{\downarrow\uparrow}_{i/j}$ are representing unpolarized and polarized quark distributions in a j-valon. Defining the unpolarized and polarized valon distributions as

$$G_{j/p} \equiv G^{\uparrow}_{j/p} + G^{\downarrow}_{j/p}\,, \qquad (2.12)$$

and

$$\delta G_{j/p} \equiv G^{\uparrow}_{j/p} - G^{\downarrow}_{j/p}\,, \qquad (2.13)$$

we will obtain

$$q_{i/p}(x,Q^2) = \sum_j \int_x^1 q_{i/j}\left(\frac{x}{y},Q^2\right) G_{j/p}(y)\frac{dy}{y}\,, \qquad (2.14)$$

and

$$\delta q_{i/p}(x,Q^2) = \sum_j \int_x^1 \delta q_{i/j}\left(\frac{x}{y},Q^2\right) \delta G_{j/p}(y)\frac{dy}{y}\,. \qquad (2.15)$$

As we can see similarly for the unpolarized case, the polarized quark distribution can be related to a polarized valon distribution.

2.3 Spin dependence of the valon distributions

The unpolarized and polarized quark distributions are respectively defined as

$$\begin{aligned}
q(x) &= q^{\uparrow}(x) + q^{\downarrow}(x)\,, \\
\delta q(x) &= q^{\uparrow}(x) - q^{\downarrow}(x)\,.
\end{aligned}$$

For the polarized parton distributions, $|\delta f(x,Q^2)|$, and the unpolarized ones, $f(x,Q^2)$, positivity requirements at low values of Q^2 imply the constraint [10, 11]

$$|\delta f(x,Q^2)| \leq f(x,Q^2)\,, \tag{2.16}$$

where $f = u, \bar{u}, d, \bar{d}, s, \bar{s}, g$. Furthermore, we have the following sum rules [11]

$$\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d} \;=\; A_3 = 1.2573 \pm 0.0028\,, \tag{2.17}$$

$$\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2(\Delta s + \Delta\bar{s}) \;=\; A_8 = 0.579 \pm 0.025\,, \tag{2.18}$$

$$\Delta\Sigma = \sum_q (\Delta q + \Delta\bar{q}) \;=\; A_8 + 3(\Delta s + \Delta\bar{s}) \equiv A_0\,, \tag{2.19}$$

where with $n = 1$, the first moment Δf is defined by

$$\Delta f(Q^2) = \int_0^1 dx \delta f(x,Q^2)\,. \tag{2.20}$$

It is helpful to consider the above reasonable theoretical constraints especially for the sea quark distributions. To determine the polarized parton distribution functions (PPDF's), the main step is to relate the polarized input densities to the unpolarized ones [11] using some intuitive theoretical argument as the guideline. We employ the general ansatz for the polarized parton distributions of ref. [11], and introduce the following equations to relate the polarized and unpolarized valon distributions

$$\delta G_{j/p}(y) = \delta\mathcal{W}_j(y) \times G_{j/p}(y)\,, \tag{2.21}$$

here j refers to U and D type valons. The functions $\delta\mathcal{W}_j(y)$ play an essential role in constructing the polarized valon distributions $\delta G_{j/p}(y)$ from unpolarized ones, as we shall explain in subsection 2.5.

Let us define the Mellin moments of any polarized U and D-valon distributions in the proton, $\delta G_{U,D/p}(y)$, as follows:

$$\Delta M_{U/p}(n) \equiv \int_0^1 y^{n-1} \delta G_{U/p}(y)\, dy\,, \tag{2.22}$$

$$\Delta M_{D/p}(n) \equiv \int_0^1 y^{n-1} \delta G_{D/p}(y)\, dy\,. \tag{2.23}$$

Using polarized valon distributions, the moments of polarized valon distributions, $\Delta M_{U/p}(n)$ and $\Delta M_{D/p}(n)$, can be easily calculated. The moments of polarized parton distributions in the proton can be written like the unpolarized case [9] as follows

$$\Delta u_v(n,Q^2) \;=\; 2\Delta M^{NS}(n,Q^2) \times \Delta M_{U/p}(n)\,, \tag{2.24}$$

$$\Delta d_v(n,Q^2) \;=\; \Delta M^{NS}(n,Q^2) \times \Delta M_{D/p}(n)\,, \tag{2.25}$$

$$\Delta\Sigma(n,Q^2) \;=\; \Delta M^{S}(n,Q^2) \times (2\Delta M_{U/p}(n) + \Delta M_{D/p}(n))\,. \tag{2.26}$$

The factor 2 in eqs. (2.24), (2.26) is due to the existence of two-U type valons.

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In the above equations, $\Delta M^{NS}(n,Q^2)$ and $\Delta M^S(n,Q^2)$ are the moments of the non-singlet and singlet sectors which describe parton distributions inside the valon. We will introduce these functions in the LO and NLO approximations in section 3. As we will see, in the LO approximation the first moment, $\Delta M^{NS}(n=1,Q^2)$ and $\Delta M^S(n=1,Q^2)$ will be equal to 1 for all values of Q^2. So, using eqs. (2.24)–(2.26) we have

$$\Delta u_v \equiv \Delta u_v(1,Q^2) = 2\Delta M_{U/p}(1)\,, \tag{2.27}$$

$$\Delta d_v \equiv \Delta d_v(1,Q^2) = \Delta M_{D/p}(1)\,, \tag{2.28}$$

$$\Delta\Sigma \equiv \Delta\Sigma(1,Q^2) = 2\Delta M_{U/p}(1) + \Delta M_{D/p}(1)\,. \tag{2.29}$$

It can be seen that in the LO approximation the first moment of the polarized quark distributions is equal to the first moment of the related polarized valon distributions. We should notice that in the NLO approximation, the first moment of the polarized quark distributions are Q^2-dependent, and the above equations are applicable only for $Q^2 = Q_0^2$.

The Σ symbol in eq. (2.29) denotes $\sum_{q=u,d,s}(q+\bar{q})$ and consequently we have the following definition of $\Delta\Sigma$

$$\Delta\Sigma(n,Q^2) = \sum_{q=u,d,s} (\Delta q(n,Q^2) + \Delta\bar{q}(n,Q^2))\,, \tag{2.30}$$

therefore the moment of polarized sea quarks can be obtained as

$$\Delta\bar{q}(n,Q^2) = (\Delta\Sigma(n,Q^2) - \Delta u_v(n,Q^2) - \Delta d_v(n,Q^2))/2f\,, \tag{2.31}$$

where f is the number of active quark flavours. If we choose $n=1$ in eq. (2.31) and use eqs. (2.27)–(2.29), we can see that for all Q^2 values in the LO and for $Q^2 = Q_0^2$ in the NLO approximations, there is no contribution to the first moment of the sea quarks. As a result, we can not precisely determine the spin components of the hadrons. This difficulty is investigated in the following subsections.

2.4 The improvement of polarized valons

To resolve the above problem, first we improve the definition of polarized valon distribution functions

$$\delta G_{j/p} \to \begin{cases} \delta\mathcal{W}_j^{'}(y) \times G_{j/p}(y) & \text{for non-singlet case,} \\ \delta\mathcal{W}_j^{''}(y) \times G_{j/p}(y) & \text{for singlet case.} \end{cases} \tag{2.32}$$

Using the above equation and according to the eqs. (2.27)–(2.29), the first moment of the polarized u, d and Σ distribution functions can be written as follows

$$\Delta u_v = 2\int_0^1 dy\,\left[\delta\mathcal{W}_U^{'}(y) \times G_{U/p}(y)\right], \tag{2.33}$$

$$\Delta d_v = \int_0^1 dy\,\left[\delta\mathcal{W}_D^{'}(y) \times G_{D/p}(y)\right], \tag{2.34}$$

$$\Delta\Sigma = \int_0^1 dy\,\left(\left[2\,\delta\mathcal{W}_U^{''}(y) \times G_{U/p}(y)\right] + \left[\delta\mathcal{W}_D^{''}(y) \times G_{U/p}(y)\right]\right). \tag{2.35}$$

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The above equations can help us to consider the constraint of eqs. (2.17)–(2.19) for the improved polarized valon model with an SU(3) flavour symmetry assumption. These constraints play the same role as in the case of the unpolarized ones, eq. (2.5), in controlling the parameter values which will appear in the polarized valon distributions.

2.5 Description of the $\mathcal{W}$ function

To define the actual y-dependence of the $\mathcal{W}'$ function, we parameterize this function as

$$\delta\mathcal{W}_j'(y) = N_j y^{\alpha_j}(1-y)^{\beta_j}(1+\gamma_j y+\eta_j y^{0.5})\,. \tag{2.36}$$

As before the subscript j refers to U and D-valons.

The motivation for choosing this functional form is that the term y^{α_j} controls the low-y behavior valon densities, and $(1-y)^{\beta_j}$ that at large values y. The remaining polynomial factor accounts for the additional medium-y values.

For $\delta\mathcal{W}_j''(y)$ we choose the following form

$$\delta\mathcal{W}_j''(y) = \delta\mathcal{W}_j'(y) \times \sum_{m=0}^{5} \mathcal{A}_m y^{\frac{m-1}{2}}\,. \tag{2.37}$$

The additional term in the above equation, ($\sum$ term), serves to control the behavior of the singlet sector at very low-y values in such a way that we can extract the sea quark contributions. Moreover, the functional form for $\delta\mathcal{W}'$ and $\delta\mathcal{W}''$ give us the best fitting χ^2 value, as will be described in section4. In these functions, all of the parameters are unknown. Using the experimental data for g_1^p [12, 13] and the Bernstein polynomials, we can fit for the unknown parameters of eqs. (2.36), (2.37).

3. The NLO moments of PPDF's and the structure function

In this section moments of the polarized parton distribution functions (PPDF's) in the valon are introduced. To calculate the Q^2 evolution of the polarized parton distributions, $\delta f(x,Q^2)$, we need to Mellin transform these functions, i.e.

$$\delta f^n(Q^2) = \int_0^1 x^{n-1}\delta f(x,Q^2)dx\,. \tag{3.1}$$

The evolution of anomalous dimensions can be obtained by Mellin transformation of splitting functions, and admit an expansion in powers of the running coupling constant $\alpha_s(Q^2)$

$$\delta d_{NS\pm} = \frac{\alpha_s(Q^2)}{2\pi}\delta d_{qq}^{(0)}(x) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 \delta d_{NS\pm}^{(1)}(x) + \cdots\,, \tag{3.2}$$

whose detailed n-dependence has been specified in ref. [10]. Here $\delta d_{qq}^{(0)}$ is unique whereas all higher anomalous dimensions are scheme-dependent.

The leading and next-to-leading order solutions of the renormalization group equation for the polarized moments can be expressed entirely in terms of the evolution parameter

$$L(Q^2) \equiv \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\,. \tag{3.3}$$

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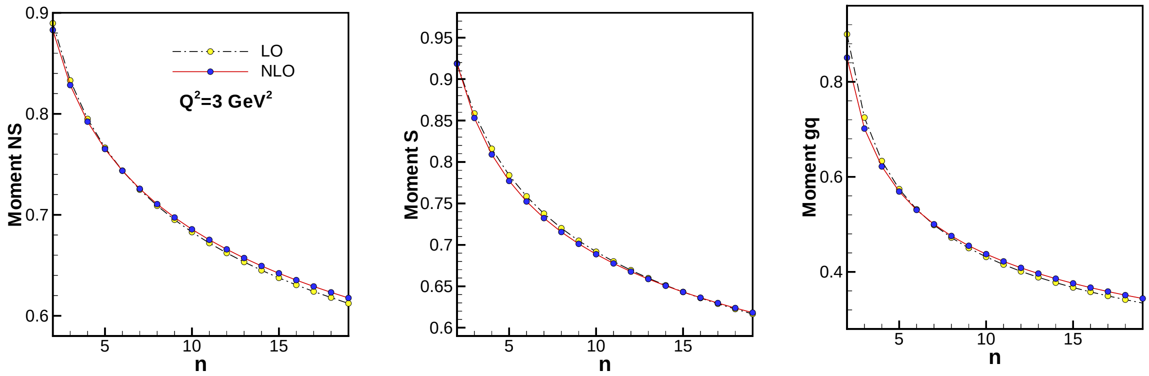

Figure 1: The contribution of moments NS, S and gq from eqs. (3.4), (3.6) as a function of n at $Q^2 = 3\,\mathrm{GeV}^2$. Dashed-dotted line and solid line indicate respectively the LO and the NLO approximations.

The non-singlet (NS) moments evolve according to

$$\Delta M^{NS\pm}(n,Q^2) = \left(1 - \frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{2\pi}\left(\delta d_{NS\pm}^{(1)n} - \frac{2\pi b'}{b}\delta d_{qq}^{(0)n}\right)\right) L^{\delta d_{qq}^{(0)n}}\,, \tag{3.4}$$

and the NLO running coupling constant is given by

$$\alpha_s(Q^2) \cong \frac{1}{b\log(Q^2/\Lambda^2_{\overline{MS}})} - \frac{b'}{b^3}\frac{\ln\left(\ln(Q^2/\Lambda^2_{\overline{MS}})\right)}{\left(\ln(Q^2/\Lambda^2_{\overline{MS}})\right)^2}\,, \tag{3.5}$$

where $b = \frac{33-2f}{12\pi}$ and $b' = \frac{153-19f}{24\pi^2}$. In the above equations, we choose $Q_0 = 1\,\mathrm{GeV}^2$ as a fixed parameter and Λ is an unknown parameter which can be obtained by fitting to experimental data.

The evolution in the flavor singlet and the gluon sector of the moments are governed by a 2×2 anomalous dimension matrix, with the following explicit solution

$$\begin{pmatrix}\Delta M^S(n,Q^2)\\ \Delta M^{gq}(n,Q^2)\end{pmatrix} = \left(L^{\delta\hat{d}^{(0)n}} + \frac{\alpha_s(Q^2)}{2\pi}\hat{U}L^{\delta\hat{d}^{(0)n}} - \frac{\alpha_s(Q_0^2)}{2\pi}L^{\delta\hat{d}^{(0)n}}\hat{U}\right)\begin{pmatrix}1\\1\end{pmatrix}, \tag{3.6}$$

where ΔM^{gq} indicates the spin dependent quark-to-gluon evolution function. All the associated functions in the above equation have been defined in ref. [14].

Using eqs. (3.4), (3.6), the moments of NS, S and gq as a function of n are calculable. These moments in the LO and NLO approximations at $Q^2 = 3\,\mathrm{GeV}^2$ are presented in figure 1.

Determination of the moments of parton distributions in a proton can be done strictly through the moments of the polarized valon distributions. The parton distributions in the x-space which will be calculated, are δu_v, δd_v, $\delta\Sigma$ and δg. The moments of these distributions are denoted respectively by: $\Delta u_v(n,Q^2)$, $\Delta d_v(n,Q^2)$, $\Delta\Sigma(n,Q^2)$ and $\Delta g(n,Q^2)$. Therefore, the moments of the polarized u and d-valence quark in a proton are convolutions of two moments:

$$\Delta u_v(n,Q^2) = 2\Delta M^{NS}(n,Q^2)\times \Delta M'_{U/p}(n)\,, \tag{3.7}$$

$$\Delta d_v(n,Q^2) = \Delta M^{NS}(n,Q^2)\times \Delta M'_{D/p}(n)\,, \tag{3.8}$$

The moment of the polarized singlet distribution (Σ) is as follows:

$$\Delta\Sigma(n,Q^2) = \Delta M^S(n,Q^2)(2\Delta M''_{U/p}(n) + \Delta M''_{D/p}(n)) \,. \tag{3.9}$$

For the leading and next-to-leading terms in eqs. (3.4), (3.6), for the NS and S sectors, we can see that for all values of Q^2 in the LO and for $Q^2 = {Q_0}^2$ in the NLO, the first moments of $\Delta M^{NS}(1,Q^2)$ and $\Delta M^S(1,Q^2)$ will be equal to one, and consequently the first moment of the polarized u and d quarks are respectively proportional to the first moment of the polarized U and D type valon distributions.

For the gluon distribution we have

$$\Delta g(n,Q^2) = \Delta M^{gq}(n,Q^2)\left(2\Delta M'_{U/p}(n) + \Delta M'_{D/p}(n)\right), \tag{3.10}$$

where $\Delta M^{gq}(n,Q^2)$ is the quark-to-gluon evolution function, given in eq. (3.6).

The NLO moment contribution of $g_1(x,Q^2)$ [14] has the following form

$$g_1(n,Q^2) = \frac{1}{2}\sum_q e_q^2\left\{\left(1+\frac{\alpha_s}{2\pi}\delta C_q^n\right)\left[\Delta q(n,Q^2)+\Delta\bar{q}(n,Q^2)\right] + \frac{\alpha_s}{2\pi}2\delta C_g^n\Delta g(n,Q^2)\right\}, \tag{3.11}$$

where $\Delta q(n,Q^2) = \Delta q_v(n,Q^2) + \Delta\bar{q}(n,Q^2)$, $\Delta\bar{q}(n,Q^2)$ and $\Delta g(n,Q^2)$ are moments of the polarized parton distributions in a proton. δC_q^n, δC_g^n are also the n-th moments of spin-dependent Wilson coefficients given by

$$\delta C_q^n = \frac{4}{3}\left[-S_2(n) + (S_1(n))^2 + \left(\frac{3}{2} - \frac{1}{n(n+1)}\right)S_1(n) + \frac{1}{n^2} + \frac{1}{2n} + \frac{1}{n+1} - \frac{9}{2}\right], \tag{3.12}$$

and

$$\delta C_g^n = \frac{1}{2}\left[-\frac{n-1}{n(n+1)}(S_1(n)+1) - \frac{1}{n^2} + \frac{2}{n(n+1)}\right], \tag{3.13}$$

with $S_k(n)$ defined as in ref. [14].

So far the moments of the polarized parton distributions have been determined and the moments of the polarized proton structure function in the NLO approximation can be obtained by inserting the required distribution functions in eq. (3.11). There are some unknown parameters in eqs. (2.36), (2.37) and eq. (3.5) which they also appear in eqs. (3.7)–(3.10). It is obvious that the final form for $g_1(n,Q^2)$ involves the total of 17 unknown parameters. If the parameters can be obtained then the computation of all moments of the polarized parton distributions and the structure function, $g_1(n,Q^2)$, are possible.

4. QCD analysis method

4.1 The Bernstein averages of moments

In the phenomenological investigations of the structure functions, for a given value of Q^2, only a limited number of experimental points, covering a partial range of values of x, are available. Therefore, one cannot directly determine the moments. A method devised to deal with this situation is to take averages of the structure function weighted by suitable polynomials. We can compare theoretical predictions with the experimental results for the Bernstein averages, which are defined by [15]

$$g_{n,k}(Q^2)\equiv\int_0^1 dx p_{nk}(x) g_1(x,Q^2)\,, \tag{4.1}$$

where $p_{n,k}(x)$ are the Bernstein polynomials,

$$p_{n,k}(x)=\frac{\Gamma(n+2)}{\Gamma(k+1)\Gamma(n-k+1)}x^k(1-x)^{n-k}\,, \tag{4.2}$$

and are normalized to unity, $\int_0^1 dx p_{n,k}(x)=1$. Therefore, the integral (4.1) represents an average of the function $g_1(x,Q^2)$ in the region $\bar{x}_{n,k}-\frac{1}{2}\Delta x_{n,k}\leq x\leq \bar{x}_{n,k}+\frac{1}{2}\Delta x_{n,k}$ where $\bar{x}_{n,k}$ is the average of x which is very near to the maximum of $p_{n,k}(x)$, and $\Delta x_{n,k}$ is the spread of $\bar{x}_{n,k}$. The key point is that values of g_1 outside this interval have a small contribution to the above integral, as $p_{n,k}(x)$ decreases to zero very quickly. In order to ensure the equivalence of the integral (4.1) to the same integral in the range $x_1=\bar{x}_{n,k}-\frac{1}{2}\Delta x_{n,k}$ to $x_2=\bar{x}_{n,k}+\frac{1}{2}\Delta x_{n,k}$, we have to use the normalization factor of $\int_{x_1}^{x_2} dx p_{n,k}(x)$ in the denominator of eq. (4.1) which obviously is not equal to 1.

By a suitable choice of n, k we manage to adjust the region where the average is peaked to that in which we have experimental data [12, 13]. Using the binomial expansion in eq. (4.2), it follows that the averages of g_1 with $p_{n,k}(x)$ as weight functions, can be obtained in terms of odd and even moments,

$$g_{n,k}=\frac{(n-k)!\Gamma(n+2)}{\Gamma(k+1)\Gamma(n-k+1)}\sum_{l=0}^{n-k}\frac{(-1)^l}{l!(n-k-l)!}\int_0^1 x^{((k+l+1)-1)}g_1(x,Q^2)dx\,, \tag{4.3}$$

and using the definition of Mellin moments of any hadron structure function we have

$$g_{n,k}=\frac{(n-k)!\Gamma(n+2)}{\Gamma(k+1)\Gamma(n-k+1)}\sum_{l=0}^{n-k}\frac{(-1)^l}{l!(n-k-l)!}g_1((k+l)+1,Q^2)\,. \tag{4.4}$$

We can only include a Bernstein average, $g_{n,k}$, if we have experimental points covering the whole range $[\bar{x}_{n,k}-\frac{1}{2}\Delta x_{n,k},\bar{x}_{n,k}+\frac{1}{2}\Delta x_{n,k}]$ [16]. This means that with the available experimental data we can only use the following 41 averages,

$$g_{2,1}^{(\mathrm{exp})}(Q^2), g_{2,2}^{(\mathrm{exp})}(Q^2),\ldots,g_{13,10}^{(\mathrm{exp})}(Q^2)\,.$$

Another restriction which we assume here, is to ignore the effects of moments with high order n which do not strongly constrain the fits. To obtain these experimental averages from the E143 and SMC data [12, 13], we fit $xg_1(x,Q^2)$ for each bin in Q^2 separately, to the convenient phenomenological expression

$$xg_1^{(\mathrm{phen})}=\mathcal{A}x^{\mathcal{B}}(1-x)^{\mathcal{C}}\,. \tag{4.5}$$

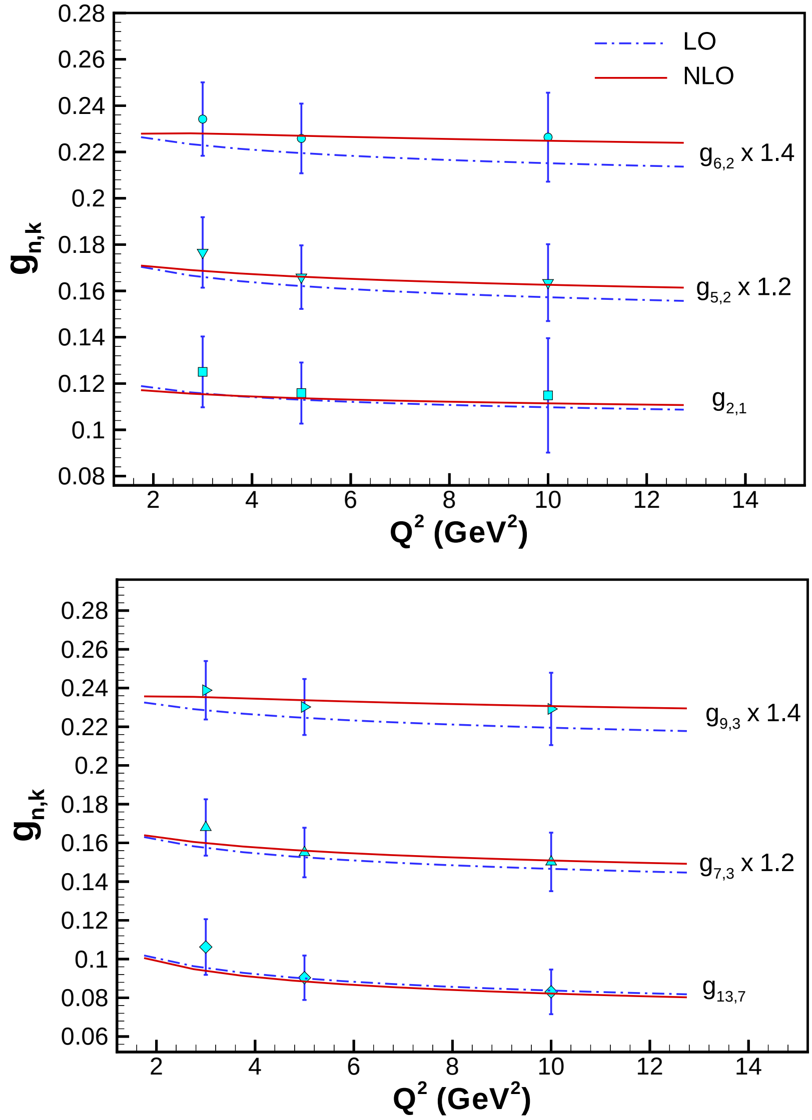

Figure 2: Fit to xg_1^p using the Bernstein averages. Dashed-dotted line and solid line indicate respectively the LO and the NLO approximations.

$Q^2\,\mathrm{GeV}^2$	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{C}$
3	0.274	0.876	1.212
5	0.339	0.911	1.724
10	0.342	0.842	1.932

Table 1: Numerical values of fitting $\mathcal{A}, \mathcal{B}, \mathcal{C}$ parameters in eq. (4.5).

This form ensures zero values for xg_1 at $x = 0$, and $x = 1$. In table 1 we have presented the numerical values of $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ for $Q^2 = 3, 5, 10\,\mathrm{GeV}^2$. A theoretical justification of eq. (4.5) may be found in ref. [17]. Using eq. (4.5) with the fitted values of $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ one can then compute $g_{n,k}^{(\mathrm{exp})}(Q^2)$ using eq. (4.3), in terms of Gamma functions. Some sample experimental Bernstein averages are plotted in figure 2 in the LO and NLO approximations. The errors in the $g_{n,k}^{(\mathrm{exp})}(Q^2)$ correspond to allowing the E143 and SMC data for xg_1 to vary within the experimental error bars, including the experimental systematic and statistical errors [12, 13]. We have only included data for

$Q^2 \geq 3\,\mathrm{GeV}^2$, this has the merit of simplifying the analysis by avoiding evolution through flavor thresholds.

4.2 QCD fits to the Bernstein averages for g_1

Using eq. (4.4), the 41 Bernstein averages $g_{n,k}(Q^2)$ can be written in terms of odd and even moments $g_1(n, Q^2)$:

$$
\begin{aligned}
g_{2,1}(Q^2) &= 6\left(g_1(2,Q^2) - g_1(3,Q^2)\right), \\
g_{2,2}(Q^2) &= 3\left(g_1(3,Q^2)\right), \\
g_{3,1}(Q^2) &= 12\left(g_1(2,Q^2) - 24 g_1(3,Q^2) + 12 g_1(4,Q^2)\right), \\
g_{3,2}(Q^2) &= 12\left(g_1(3,Q^2) - g_1(4,Q^2)\right), \\
&\vdots \\
g_{13,10}(Q^2) &= 4004.0\left(g_1(11,Q^2) - g_1(14,Q^2)\right) - \\
&\quad -12012\left(g_1(12,Q^2) - g_1(13,Q^2)\right). \qquad (4.6)
\end{aligned}
$$

The unknown parameters according to eqs. (2.36), (2.37) will be, N_U, α_U, β_U, $\ldots$, N_D, α_D, β_D, $\ldots$, $\mathcal{A}_0$, $\mathcal{A}_1$, $\ldots$, $\mathcal{A}_6$ and finally $\Lambda_{\overline{MS}}$ from eq. (3.5). Thus, there are 17 parameters to be simultaneously fitted to the experimental $g_{n,k}(Q^2)$ averages. Using the CERN subroutine MINUIT [18], we defined a global χ^2 for all the experimental data points and found an acceptable fit with minimum $\chi^2/\mathrm{d.o.f.} = 1.26$ in the LO case and $\chi^2/\mathrm{d.o.f.} = 0.94$ in the NLO case with the standard error of order 10^{-3}. The best fit is indicated by some sample curves in figure 2. The minimum χ^2 values for all 17 fitting parameters are listed in table 2.

Now from eqs. (2.36), (2.37), and using (2.32), we are able to determine the improved polarized U and D valon distributions in the proton. In figure 3 we have plotted the LO and NLO approximation results of $\sum_j y\delta G_{j/p}$ for the non-singlet and singlet polarized valon distributions as a function of y. In this summation j runs over U, U, D valons.

LO

j	N_j	α_j	β_j	γ_j	η_j
U	0.002	−2.3789	−1.7518	11.0804	−1.4629
D	−0.005	−1.5465	−1.8776	8.5042	−0.8608

$\mathcal{A}_0$	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	$\Lambda_{\overline{MS}}$ (MeV)
0.00043	0.2954	−6.9134	30.9851	−39.7383	16.4605	203

NLO

j	N_j	α_j	β_j	γ_j	η_j
U	0.0038	−2.1501	−0.8859	10.6537	−0.1548
D	−0.0046	−1.5859	−1.5835	9.6205	−0.8410

$\mathcal{A}_0$	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$	$\mathcal{A}_4$	$\mathcal{A}_5$	$\Lambda_{\overline{MS}}$ (MeV)
−0.0025	−3.1148	15.8114	−21.1500	10.5025	−0.9162	235

Table 2: Numerical values of fitting parameters for the best fit of figure 2 in the LO and NLO approximations.

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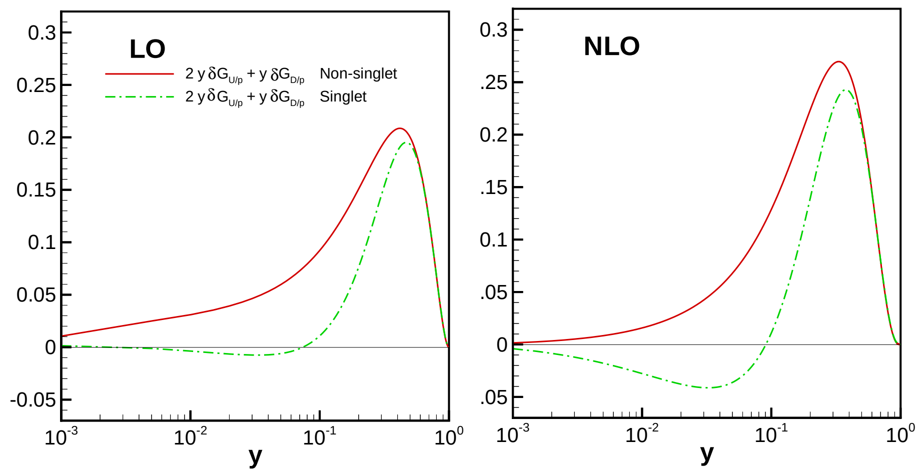

Figure 3: The plots of total polarized valon distributions in the proton, $\sum_j y\delta G_{j/p}$, for non-singlet and singlet polarized valon distributions as a function of y where j runs over U, U, D valons.

Since the polarized valon distributions are determined, we can obtain all polarized quark distributions in the proton using the convolution integral of eq. (2.15).

5. Polarized parton distributions in the x-space

Polarized valon distributions in the proton have been calculated in subsection 2.4 and section 4. Using the polarized parton structure functions in a valon, it is then possible to extract polarized parton structure functions in a proton. To obtain the z-dependence of parton distributions from the n-dependent exact analytical solutions in the Mellin-moment space, one has to perform a numerical integral in order to invert the Mellin-transformation

$$\delta f^k(z, Q^2) = \frac{1}{\pi}\int_0^\infty dw Im[e^{i\phi} z^{-c-we^{i\phi}} \Delta M_k(n = c + we^{i\phi}, Q^2)]\,, \tag{5.1}$$

where the contour of the integration lies to the right of all singularities of $\Delta M_k(n = c + we^{i\phi}, Q^2)$ in the complex n-plane. For all practical purposes one may choose $c \simeq 1, \phi = 135°$ and an upper limit of integration, for any Q^2, of about $5 + 10/\ln z^{-1}$, instead of ∞, which guarantees stable numerical results [19, 20].

Inserting in eq. (5.1) the three functions for $k = NS, S, gq$, i.e. $\Delta M^{NS}(n, Q^2)$, $\Delta M^S(n, Q^2)$ and $\Delta M^{gq}(n, Q^2)$ separately, we can obtain all polarized parton distribution functions inside the valon as a function of z, and at a fixed Q^2 value. We denote them by $\delta f^k(z = \frac{x}{y}, Q^2)$. These functions are independent of the type of valon but depend on Q^2. As was discussed in subsection 2.2, the relationship between the polarized parton distributions of a proton and the polarized parton distributions in a valon can be given by eq. (2.15) and now by using this equation, we can get the following expressions for the polarized parton distributions in a proton:

$$\delta u_v(x, Q^2) = 2\int_x^1 \delta f^{NS}\left(\frac{x}{y}, Q^2\right)\left[\delta \mathcal{W}'_U(y) \times G_{U/p}(y)\right]\frac{dy}{y}\,,$$

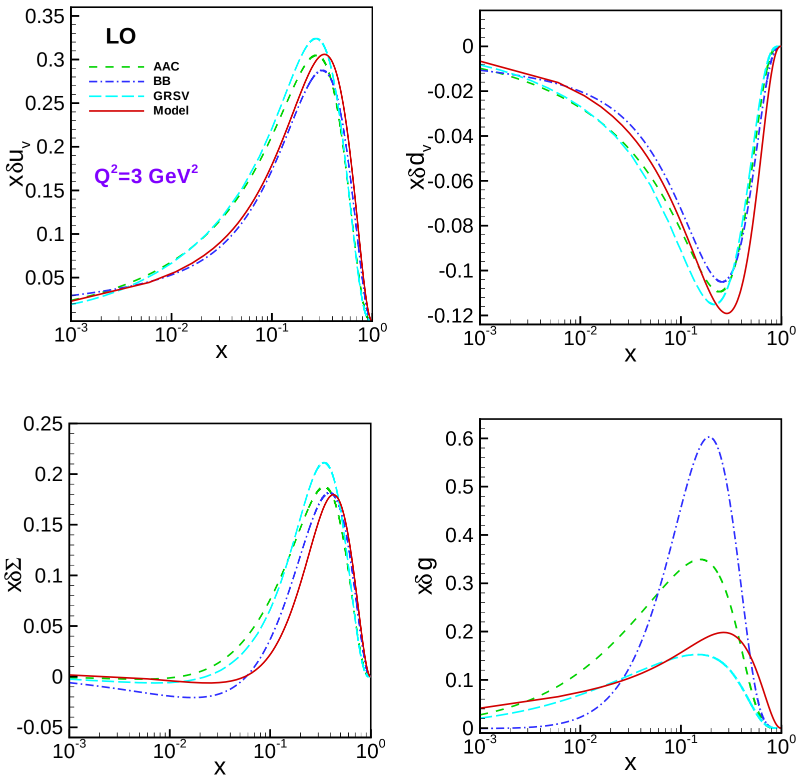

Figure 4: Polarized parton distributions in the proton at Q^2=3 GeV^2 as a function of x in the LO approximation. The solid line is our model, dashed line is the AAC model (ISET=1) [21], dashed-dotted line is the BB model (ISET=1) [22] and long-dashed line is the GRSV model (ISET=3) [23].

$$
\begin{aligned}
\delta d_v(x, Q^2) &= \int_x^1 \delta f^{NS}\left(\frac{x}{y}, Q^2\right)\left[\delta \mathcal{W}_D'(y) \times G_{D/p}(y)\right]\frac{dy}{y}\,, \\
\delta\Sigma(x, Q^2) &= \int_x^1 \delta f^{S}\left(\frac{x}{y}, Q^2\right)\Big(2\,[\delta \mathcal{W}_U''(y) \times G_{U/p}(y)] + [\delta \mathcal{W}_D''(y) \times G_{D/p}(y)]\Big)\frac{dy}{y}\,, \\
\delta g(x, Q^2) &= \int_x^1 \delta f^{gq}\left(\frac{x}{y}, Q^2\right)\Big(2\,[\delta \mathcal{W}_U'(y) \times G_{U/p}(y)] + [\delta \mathcal{W}_D'(y) \times G_{D/p}(y)]\Big)\frac{dy}{y}\,. \quad (5.2)
\end{aligned}
$$

In figure 4 we have presented the polarized parton distributions in a proton at $Q^2 = 3\,\mathrm{GeV}^2$. These distributions were calculated in the LO approximation and compared with some theoretical models [21]–[23]. In figure 5 we have presented the same distributions but in the NLO approximation.

6. Results

6.1 Polarized structure function

In the last section PPDF's were calculated and now they can be used to extract the polarized structure function g_1^p in the LO and NLO approximations.

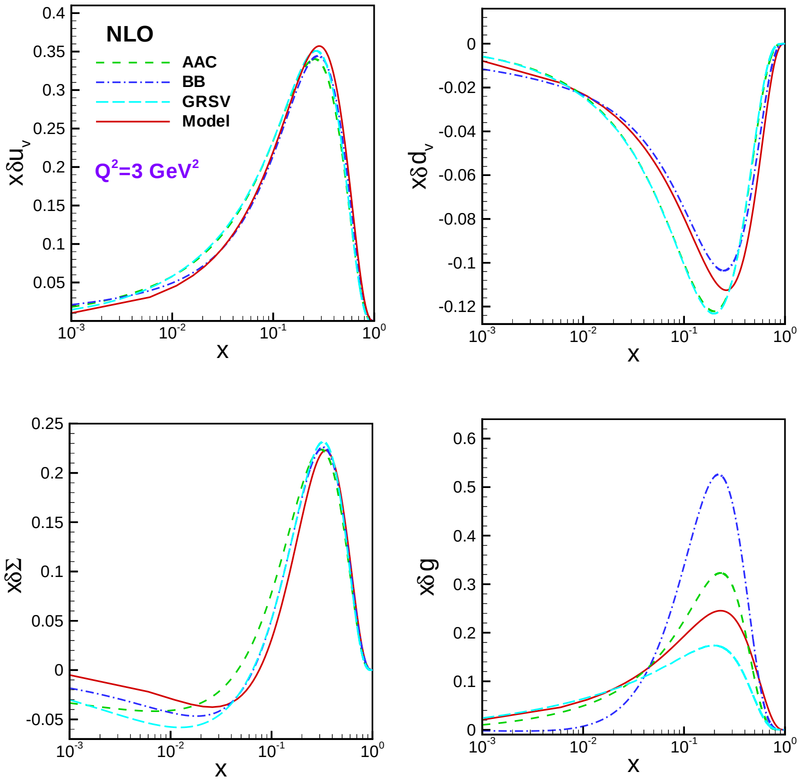

Figure 5: Polarized parton distributions in the proton at Q^2=3 GeV^2 as a function of x in the NLO approximation. The solid line is our model, dashed line is the AAC model (ISET=3) [21], dashed-dotted line is the BB model (ISET=3) [22] and long-dashed line is the GRSV model (ISET=1) [23].

According to the quark model, in the LO approximation, g_1^p can be written as a linear combination of δq and $\delta\overline{q}$ [14, 24],

$$g_1^p(x,Q^2) = \frac{1}{2}\sum_q e_q^2\left[\delta q(x,Q^2) + \delta\overline{q}(x,Q^2)\right], \tag{6.1}$$

where e_q are the electric charges of the (light) quark-flavours $q = u, d, s$. Here the sum usually runs over the light quark-flavours $q = u, d, s$, since the heavy quark contributions $(c,\ b,\ \ldots)$ could preferably be calculated perturbatively from the intrinsic light quark (u, d, s) and gluon (g) partonic-constituents of the nucleon.

Within the $\overline{MS}$ factorization scheme the NLO contributions to $g_1^p(x,Q^2)$ are finally given by [14]

$$g_1^p(x,Q^2) = \frac{1}{2}\sum_q e_q^2\left\{\delta q(x,Q^2) + \delta\bar{q}(x,Q^2) + \frac{\alpha_s(Q^2)}{2\pi}[\delta C_q\otimes(\delta q+\delta\bar{q}) + 2\delta C_g\otimes\delta g]\right\}, \tag{6.2}$$

with the convolutions defined as

$$(C\otimes q)(x,Q^2) = \int_x^1 \frac{dy}{y}C\left(\frac{x}{y}\right)q(y,Q^2)\,. \tag{6.3}$$

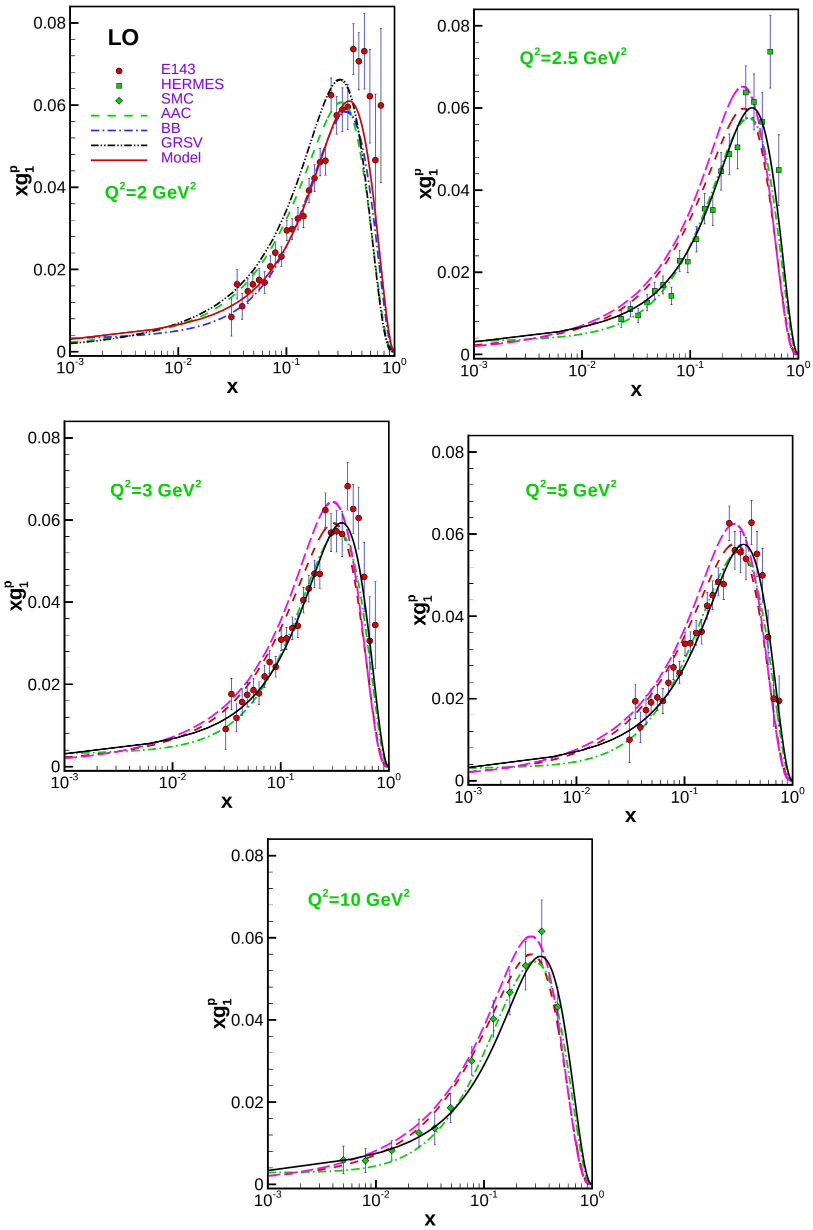

Figure 6: Polarized proton structure function xg_1^p as a function of x which is compared with the experimental data from ref. [12, 13] for different Q^2 values in the LO approximation. The solid line is our model, dashed line is the AAC model (ISET=1) [21], dashed-dotted line is the BB model (ISET=1) [22] and long-dashed line is the GRSV model (ISET=3) [23].

The polarized proton structure function, xg_1^p, in the LO and NLO approximations can now be presented. The results for xg_1^p in the LO and NLO approximations as a function of x and for some different values of Q^2 are depicted in figures 6 and 7. The comparison between different models [21]–[23] and the available experimental data [12, 13] have also been presented.

Since $\Delta(q + \overline{q})$ is the net number of right-handed quarks of flavor q inside a right-handed proton, it follows that $\frac{1}{2}\Delta\Sigma$ is a measure of how much all quark flavours contribute

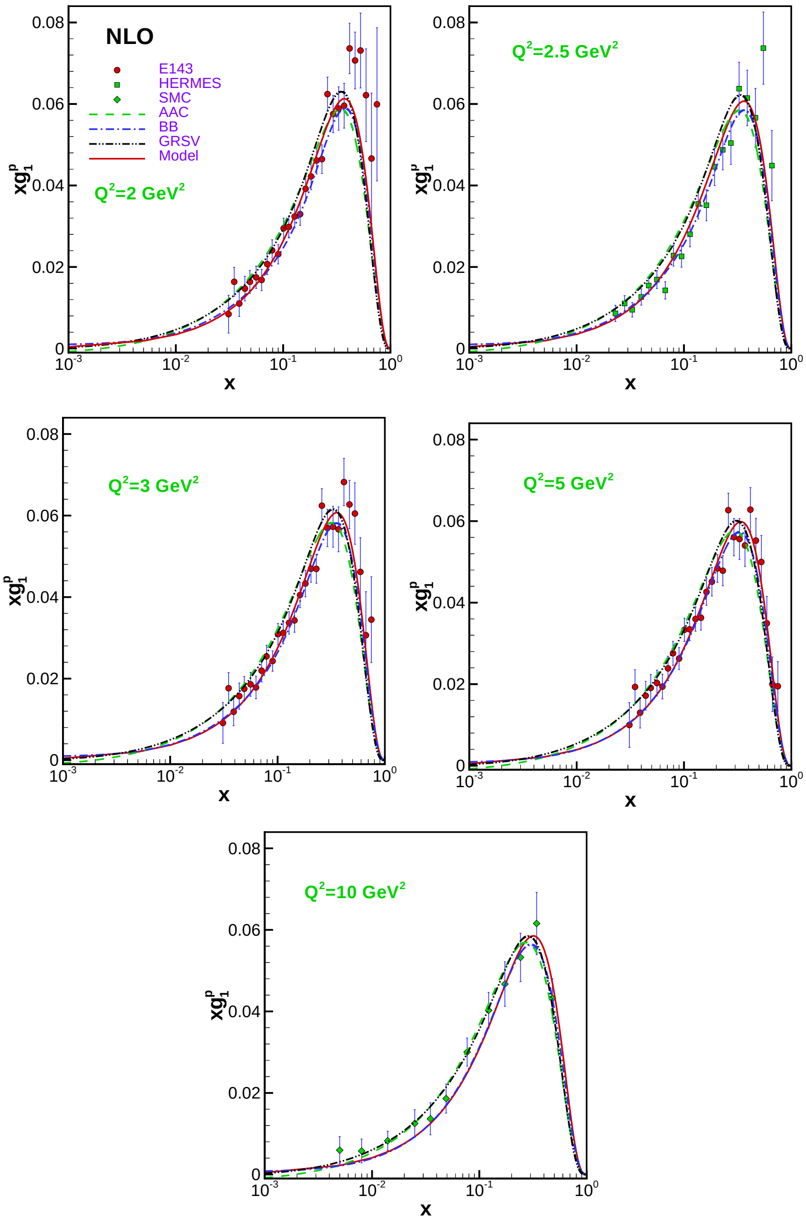

Figure 7: Polarized proton structure function xg_1^p as a function of x which is compared with the experimental data from ref. [12, 13] for different Q^2 values in the NLO approximation. The solid line is our model, dashed line is the AAC model (ISET=3) [21], dashed-dotted line is the BB model (ISET=3) [22] and long-dashed line is the GRSV model (ISET=1) [23].

to the spin of the proton. Similarly, Δg represents the total gluonic contribution to the spin of the nucleon. Using eq. (2.20), first moments of the polarized parton distributions are calculated. These results have been shown in table 3.

Finally we can define, similarly to eq. (2.20), the first moment of g_1^p (the Ellis-Jaffe sum rule) by

$$\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x, Q^2)\,. \tag{6.4}$$

Q^2 (GeV^2)	Δu_v	Δd_v	$\Delta\Sigma$	$\Delta\overline{q}$	Δg	Γ_1^p
1	0.8807	−0.3328	0.1786	−0.0615	0.5480	0.1214
3	0.8774	−0.3315	0.1724	−0.0622	0.7774	0.1242
5	0.8765	−0.3311	0.1706	−0.0624	0.8737	0.1249
10	0.8754	−0.33308	0.1688	−0.0626	0.9986	0.1258

Table 3: The first moments of polarized parton distributions, Δu_v, Δd_v, $\Delta\Sigma$, $\Delta\overline{q}$, Δg and Γ_1^p in the NLO approximation for some value of Q^2.

The results have also been given in table 3.

6.2 Spin contribution of the valons to the proton

The spin contribution of the valons to the proton can be analyzed through the valon model. The contribution of various polarized partons in a valon are calculable and by computing their first moment, the spin of the proton can be computed. In the framework of QCD the spin of the proton can be expressed in terms of the first moment of the total quark and gluon helicity distributions and their orbital angular momentum, i.e.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^p + \Delta g^p + L_z^p\,, \tag{6.5}$$

where L_z^p is the total orbital angular momentum of all the quarks and gluons.

Using eq. (5.2) the contributions of $\delta\Sigma(x,Q^2)$ and $\delta g(x,Q^2)$ in the proton can be calculated. These contributions for just one U valon in a proton can be extracted as

$$\begin{aligned}\delta\Sigma^U(x,Q^2) &= \int_x^1 \delta f^S\left(\frac{x}{y},Q^2\right)\left[\delta\mathcal{W}''_U(y)\times G_{U/p}(y)\right]\frac{dy}{y}\,,\\ \delta g^U(x,Q^2) &= \int_x^1 \delta f^{gq}\left(\frac{x}{y},Q^2\right)\left[\delta\mathcal{W}'_U(y)\times G_{U/p}(y)\right]\frac{dy}{y}\,.\end{aligned} \tag{6.6}$$

Using the above equations we can arrive at their first moments as in below

$$\begin{aligned}\Delta\Sigma^U &= \int_0^1 \delta\Sigma^U(x,Q^2)dx\,,\\ \Delta g^U &= \int_0^1 \delta g^U(x,Q^2)dx\,.\end{aligned} \tag{6.7}$$

The results for a D-valon can be obtained in a similar way. The resulting total quark and gluon helicity for a U-valon is

$$\frac{1}{2}\Delta\Sigma^U + \Delta g^U\,, \tag{6.8}$$

and for D-valon

$$\frac{1}{2}\Delta\Sigma^D + \Delta g^D\,. \tag{6.9}$$

Since each proton involves 2 U-valons and one D-valon, the total quark and gluon helicity for the proton is

$$\frac{1}{2}\Delta\Sigma^p + \Delta g^p = 2\left(\frac{1}{2}\Delta\Sigma^U + \Delta g^U\right) + \frac{1}{2}\Delta\Sigma^D + \Delta g^D\,. \tag{6.10}$$

i	$\frac{1}{2}\Delta\Sigma^i$	Δg^i	L_z^i	$\frac{1}{2}\Delta\Sigma^i + \Delta g^i + L_z^i$
U	0.091	0.440	−0.031	0.5
D	−0.077	−0.332	−0.091	−0.5
P	0.105	0.548	−0.153	0.5

(a)

i	$\frac{1}{2}\Delta\Sigma^i$	Δg^i	L_z^i	$\frac{1}{2}\Delta\Sigma^i + \Delta g^i + L_z^i$
U	0.039	0.440	0.020	0.5
D	0.011	−0.332	−0.178	−0.5
P	0.089	0.548	−0.137	0.5

(b)

Table 4: Spin contribution of valons and proton in the LO (a) and the NLO (b) approximations at $Q^2 = Q_0^2$.

With the assumption that the polarized U and D valons have spin of $+\frac{1}{2}$ and $-\frac{1}{2}$ respectively, the determination of orbital angular momentum for each valon and finally for the proton, at different values of Q^2, can be obtained from eq. (6.5). Our results at $Q^2 = Q_0^2$ for the LO and NLO approximations are gathered in table 4.

7. Conclusions

We have used the valon model to describe deep inelastic scattering. The model bridges the gap between the bound state problem and the scattering problem for hadrons. The valon distribution can serve as a model for the solution to the bound state problem. In contrast to this model, people usually use the GLAP equations to evolve the parton distributions from an initial value Q_0. The valon model which was first introduced by Hwa [1]–[7],[9], gives us a clear insight as to how to construct the hadron structure from the parton distributions. According to this model, a valon is defined as a cluster of valence quarks accompanied by a cloud of sea quarks and gluons. It can be considered as a bound state in which for instance a proton consists of three valons, two U-valons and one D-valon. This model has been applied to the unpolarized case and gives acceptable phenomenological results.

In this paper we extended the idea of the valon model to the polarized case to describe the spin dependence of the hadron structure functions. The polarized valon distribution is derived from the unpolarized valon distribution. From the phenomenological point view, we need to define weight functions to construct a functional form for the polarized valon distribution. Due to the difficulty which exists in deriving the spin contribution of the sea quarks for all values Q^2 in the LO and at $Q^2 = Q_0^2$ in the NLO approximations, we improved the weight functions and introduced two different types of them for the singlet and non-singlet cases. In deriving the polarized valon distributions some unknown parameters are present which should be determined by fitting to the experimental data. Here we have used a method similar to ref. [8], to fit QCD predictions for the moments of the g_1^p structure function to the suitably constructed Bernstein polynomial averages of the E143 and SMC experimental data.

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After calculating polarized valon distributions and all the parton distributions in a valon, we calculated the polarized parton density in a proton. The results were used to evaluate the spin components of the proton. It turns out that our results for the polarized structure functions are in good agreement with all the available experimental data of g_1^p for the proton. Our prediction for Γ_1^p, the first moment of g_1^p, is also in good agreement with the experimental data.

We hope to report in future on application of the improved valon model in describing more complicated hadron structure functions. We also hope to be able to consider the symmetry breaking of polarized sea quarks in our new calculations.

Acknowledgments

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